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Journal of Economic Theory 157 (2015) 172–187

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

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Coalitions, tipping points and the speed of evolution

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Received 25 February 2014; final version received 30 December 2014; accepted 6 January 2015

Available online 12 January 2015

Abstract

This study considers waiting times for populations to achieve efficient social coordination. Belloc and Bowles [1] conjecture that coalitional behavior will hasten such coordination. This turns out to be true when every member of the population interacts with every other member, but does not extend to more complex networks of interaction. Although it is in the interest of every player to coordinate on a single globally efficient norm, coalitional behavior at a local level can greatly slow, as well as hasten, convergence to efficiency.

JEL classification: C71; C72; C73

Keywords: Evolution; Coalition; Social norm; Reform; Conservatism; Networks

1. Introduction

A pervasive criticism of stochastic stability as a tool of equilibrium selection has been the large lengths of time it can take for perturbed adaptive processes to reach stochastically stable states (Young [25], Kandori and Mailath [9], Ellison [4]). There has been extensive study of such waiting times for coordination games when interaction is governed by a network. Results have

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1 This work was completed while the author was supported by a Discovery Early Career Researcher Award funded by the Australian Research Council (Grant Number: DE130101768).

http://dx.doi.org/10.1016/j.jet.2015.01.003
0022-0531/

	A	B
A	α, α	$0, 0$
B	$0, 0$	$1, 1$

Fig. 1. A two player coordination game, $\alpha > 1$.

been found to depend on network topology, network size, and whether the small error probability limit is analyzed or error probabilities are kept fixed (Ellison [5], Young [26,27], Montanari and Saberi [13], Kreindler and Young [11]).

This paper examines the effect of coalitional behavior on expected waiting times for processes to reach stochastically stable states. We focus on two action coordination games with one efficient action A, and one inefficient action B, as illustrated in Fig. 1. The relative efficiency of the efficient action to the inefficient action is given by the parameter α. The set of players with whom any given player interacts is governed by an underlying network. A stochastically stable state is the state in which every player plays the efficient action (Peski [21]). The waiting time for the process to reach this state can thus be understood as the delay before a society converges to an efficient social norm. In line with the theoretical predictions of Olson [20] and much of the subsequent literature on collective action, we are particularly interested in the effect of joint strategic switching by coalitions which are small relative to the total population size.[2]

Two possible effects of coalitional behavior are discovered, a *reforming* effect and a *conservative* effect. For any network and high values of α, we observe a reforming effect: convergence to the long run equilibrium is much faster when coalitional behavior is allowed. Less obviously, for some networks and low values of α, there is a conservative effect: convergence to the long run equilibrium is much slower in the presence of coalitional behavior. These effects, taken singly or together, imply that coalitional behavior increases the sensitivity of convergence speeds to the relative efficiency of competing norms. Several network types display tipping point effects. For values of α below some threshold $\underline{\alpha}$, coalitional behavior has a conservative effect. For values of α above some threshold $\bar{\alpha}$, coalitional behavior has a reforming effect. In some instances $\underline{\alpha}$ and $\bar{\alpha}$ take the same value. The principal results of the analysis are as follows:

(i) For any network, a reforming effect is observed for large enough α. Furthermore, for any α, a reforming effect is observed if large enough coalitions can form.
(ii) For some networks and coalition sizes, a conservative effect is observed for small enough α.
(iii) The notion of a *parochial* set of players is defined recursively, building outwards from some core players who are completely isolated from the network outside of the parochial set. It is shown that parochial sets of players are the only sets which are immune to conservative effects for any α and coalition size.
(iv) If all Nash equilibria for a network involve every player choosing the same action, then there cannot be a conservative effect for any α and coalition size. This set of networks includes the ring network and the complete network. This confirms the hypothesis of Belloc and Bowles [1] that coalitional behavior will speed convergence to efficient social norms in population games, which are equivalent to interaction on the complete network.

[2] See also Poteete and Ostrom [22]. There also exist important provisos to such predictions (Chamberlin [2]), particularly in the presence of punishment (Mathew and Boyd [12], Hwang [7]).

(v) We characterize sets of players which are immune to 'contagion' (in the sense of Morris [14]) by the efficient action. In the absence of coalitional behavior, Morris [14] gives an external stability condition: such a set of players must be sufficiently insular. In the presence of coalitional behavior, there is also an internal stability condition: the set must not contain small groups of players which are insular enough to profitably coordinate a switch to the efficient action.

The paper contributes to the small but growing literature on coalitional behavior in perturbed evolutionary models. Newton [16] introduces a model of *coalitional stochastic stability* in which the 'errors' in the dynamic process are actually small probabilities of payoff improving behavior by coalitions of players. Sawa [23] adapts coalitional stochastic stability for logit-style dynamics. The model of Sawa [23] also features coalitional behavior as part of the unperturbed dynamic. Serrano and Volij [24] and Newton [17] do similarly, applying stochastic stability to models of coalitional recontracting. Matching models such as those found in Jackson and Watts [8], Klaus, Klijn and Walzl [10] and Newton and Sawa [19], in which coalitions are pairs of recontracting agents, also fall into this category.

The paper is organized as follows. Section 2 gives the model. Section 3 introduces the ideas of the paper via an example. Section 4 gives some general results for the unperturbed process. Section 5 studies conservative effects. Section 6 concludes. All proofs not in the main body of the text are given in Appendix A.

2. Model

Let N be a finite set of players. Players are arranged in a network, which we represent as a graph $\mathbf{g}$, where $g_{ij}=1$ if there exists an edge between players i and j, and $g_{ij}=0$ otherwise. We assume that the graph is undirected: $g_{ij}=g_{ji}$. Set $g_{ii}=0$ for all $i\in N$. The network will affect players in two ways:

(i) The network determines the structure of payoffs, in particular which players' actions impose externalities on other players.
(ii) The network mediates joint action by *coalitions* of players.

Let $x_i^t\in\{A,B\}$ denote player i's action at time t. Let $x_S^t=(x_i^t)_{i\in S}$ denote the action profile of all players in $S\subseteq N$ at time t. In the absence of a subscript, $x^t:=x_N^t$. Let x_i and x_S denote representative actions and action profiles respectively. Let N_i denote the set of *neighbors* of player i: $N_i=\{j\in N: g_{ij}=1\}$. For $S\subseteq N$, let $N_S=(\bigcup_{i\in S}N_i)\setminus S$. Payoffs of player i in period t are given by:

$$u_i(x^t)=\sum_{j\in N_i}\delta(x_i^t,x_j^t)$$

where:

$$\delta(A,A)=\alpha>1;\qquad \delta(A,B)=\delta(B,A)=0;\qquad \delta(B,B)=1.$$

That is, the players play a coordination game on the network. If a player chooses action B, his payoff is the number of his neighbors who play action B. If a player chooses action A, his payoff is the number of his neighbors who play action A multiplied by some constant α which is strictly greater than 1. Effectively, the players play their chosen action against each of their neighbors in the game in Fig. 1. The model can be understood as a threshold model, with $1/(\alpha+1)$ being the proportion of a player's neighbors who must play A for the player to want to play A.[3]

The underlying dynamic process of this paper is one in which coalitions of players adjust their actions in a coordinated manner. The sets of players which can do this are determined by the underlying network.

Definition 1. A coalition of players $S \subseteq N$ is *feasible* in g, denoted $S|_g$ if and only if for all $i, j \in S$ there exists $\{s_m\}_{m=1}^{m=l}$ such that $s_1 = i$, $s_l = j$, and $s_m \in S$, $g_{s_m s_{m+1}} = 1$ for $m < l$.

That is, S is a feasible coalition if and only if S is a singleton set or there is a path between any two players in S that only uses edges between players in S. That is, players in a feasible coalition either directly interact with one another, or have interactions mediated by other players in the coalition. Another way of stating this is that the network restricted to players in S forms a connected subgraph. It is assumed that N is a feasible coalition: $\mathbf{g}$ is a connected network. This is without loss of generality: if the network comprised more than one component, analysis of each component would proceed independently of the other components.

When a coalition chooses its actions, we mandate that it chooses a *better response*. That is, players in the coalition adjust their actions in a coordinated manner such that no member of the coalition loses from the adjustment. Define the set of better responses for a set of players S:

$$A_S(x^t) := \{x_S : u_i(x_S, x^t_{N\setminus S}) \geq u_i(x^t)\ \forall i \in S\}.$$

Let $G_{A_S(x^t)}(.)$ be a probability distribution over $A_S(x^t)$. $G_{A_S(x^t)}(.)$ will determine the actions chosen by a coalition S when it is called upon to better respond. We assume full support on the set of better responses.

Assumption 1. Each $G_{A_S(x^t)}(.)$ has full support on $A_S(x^t)$.

We are particularly interested in the effect of coalitional behavior by coalitions which are small relative to the total size of the population. It is natural to assume that there are limits to how large a coalition can be. Such a limit could be a consequence of higher costs of communication for larger coalitions. The approach taken here is to limit the maximum number of players in a coalition. Let $\mathcal{N}(k)$ be the set of feasible coalitions of size k or smaller:

$$\mathcal{N}(k) = \{S \subseteq N : (S|_g \text{ and } |S| \leq k)\}.$$

For given k, let $F_k(.)$ be a distribution on $\mathcal{N}(k)$. $F_k(.)$ will determine which coalition gets the opportunity to update its actions in any given period. The process is one of asynchronous updating: only one coalition at a time updates its actions.

Assumption 2. $F_k(.)$ has full support on $\mathcal{N}(k)$.

The process of strategy updating is constructed as follows. Each period, a coalition S is chosen according to $F_k(.)$. The coalition decides on an intended new action profile for its members. Denote this intended action profile by y_S^{t+1}. This profile is chosen from the set of better responses $A_S(x^t)$:

3 Action A is p-dominant for any $p > 1/(\alpha+1)$ under the definition of Morris et al. [15].

$$y_S^{t+1} \sim G_{A_S(x^t)}(.).$$

Following the decision on which actions to take, each player will play his intended action. This is the unperturbed dynamic. A perturbed dynamic is generated by considering the possibility that a player makes a mistake when attempting to play the action he intends to play. Each player in the coalition, independently of the other players, with a small probability ε makes an *error* and chooses an action at random. That is, independently for each $i \in S$:

$$\text{With probability } 1-\varepsilon: \quad x_i^{t+1} = y_i^{t+1}$$
$$\text{With probability } \varepsilon: \quad x_i^{t+1} \sim U\big[\{A, B\}\big].$$

Finally, all players who are not part of the chosen coalition for period t do not update their actions. For all $i \in N \setminus S$:

$$x_i^{t+1} = x_i^t$$

So the change in the action profile is determined by a Markov process, $\Phi_{k,\alpha,\varepsilon}$, on state space $X := \{A, B\}^{|N|}$, with transition probabilities $P_{k,\alpha,\varepsilon}(.,.)$ derived from the above description of the process. Let $P^t_{k,\alpha,\varepsilon}(.,.)$ denote the t-step Markov transition probabilities. Each period, this process involves feasible coalitions of players changing their strategies in a payoff improving manner. When choosing his strategy, each player in the coalition independently makes an error with probability ε and chooses an action at random.[4] The process with $\varepsilon = 0$ corresponds to an unperturbed dynamic in which players do not make errors.

Note that for $\varepsilon > 0$, $\Phi_{k,\alpha,\varepsilon}$ is irreducible and aperiodic and therefore has a unique invariant measure $\pi_{k,\alpha,\varepsilon}$ which is ergodic. Denote the expected time for the process $\Phi_{k,\alpha,\varepsilon}$ to reach state y starting from x by $W_{k,\alpha,\varepsilon}(x, y)$.

Definition 2.

$$\tau_y = \min\big\{t \geq 0 : \Phi^t_{k,\alpha,\varepsilon} = y\big\}; \qquad W_{k,\alpha,\varepsilon}(x, y) = \mathbb{E}\big[\tau_y \big| \Phi^0_{k,\alpha,\varepsilon} = x\big]$$

The focus of the paper is on $W_{k,\alpha,\varepsilon}(B^{|N|}, A^{|N|})$, the expected time for the process to move from an inefficient social norm in which every player plays B to an efficient social norm in which every player plays A. We shall occasionally be interested in the set of *stable states*: absorbing states under the process with $\varepsilon = 0$. Denote this set $\Lambda_{k,\alpha}$.

$$\Lambda_{k,\alpha} := \big\{x \in X : P_{k,\alpha,0}(x, x) = 1\big\}$$

For expositional brevity, we avoid the existence of absorbing cycles under the process with $\varepsilon = 0$ by making the following assumption which holds for generic α.

Assumption 3. $\forall z \in \mathbb{N}_+, z \leq \max_{i\in N} |N_i|, \alpha z \notin \mathbb{N}_+$.

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[4] This describes errors in *implementation*. If errors were instead made in the process by which a coalition *chooses* its actions, then errors within a coalition could be perfectly correlated and different results would obtain. However, remarkably, even if a probability ε event leads *all* the members of a coalition to make mistakes, conservative effects are still possible. Some correlation between errors in the process is also fine: the results of the paper can be appropriately restated. See Newton and Angus [18] for examples.

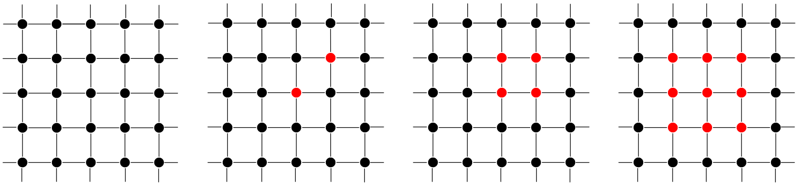

Fig. 2. Square lattice in various states. Black vertices play B, red vertices play A. (For interpretation of the colors in this figure, the reader is referred to the web version of this article.)

Throughout the paper, for functions $f(\varepsilon)$, $g(\varepsilon)$, the notation $f(\varepsilon) \in O(g(\varepsilon))$ denotes that $f(\varepsilon)$ is asymptotically (as $\varepsilon \to 0$) bounded above by some multiple of $g(\varepsilon)$. $f(\varepsilon) \in \Theta(g(\varepsilon))$ denotes that $f(\varepsilon) \in O(g(\varepsilon))$ and that $f(\varepsilon)$ is also bounded below by some multiple of $g(\varepsilon)$.

3. Coalitional effects on the square lattice

Consider the square lattice with von Neumann neighborhoods (each player neighbors other players at unit distance under the taxicab metric), pictured in Fig. 2.

Consider the benchmark case without coalitional behavior, $k = 1$, $\alpha < 3$. We have that $W_{1,\alpha,\varepsilon}(B^{|N|}, A^{|N|}) \in \Theta(\varepsilon^{-2})$. To see this, consider that two errors (Fig. 2(ii)) are necessary to move to a state C_2 (Fig. 2(iii)) in which a block of four players play A and every other player plays B. From C_2, it takes a single error to move to a state such as C_3 (Fig. 2(iv)) in which a larger block of players plays A. However, at least one error is required to move backwards from C_2 to $B^{|N|}$. That is, the probability of moving to a state in which a larger block of players plays A *conditional on* leaving C_2 is of order 1. This means that the waiting time until $A^{|N|}$ is reached involves the wait for the initial two errors of order ε^{-2}, followed by subsequent waits of order ε^{-1}. These terms combine additively and so the wait for the initial two errors dominates as ε becomes small.

The preceding argument is a slight adaptation of Ellison [5]. When $k = 1$ and $\alpha < 3$, due to C_2 being an absorbing state of the process with $\varepsilon = 0$, the probability of transiting to C_3 is of the same order of magnitude as the probability of returning to $B^{|N|}$. Put differently, once C_2 is reached, the expected time spent there is of order ε^{-1}, and over that length of time, a transition to C_3 is not a rare event. Contrast this with a situation in which an ε^2 event and an ε event have to occur relatively close together in time. This is equivalent to a single ε^3 event having to occur, the expected waiting time for which is $\Theta(\varepsilon^{-3})$.

Now consider $k = 4$, coalitions including up to four players can form. First, consider the case $\alpha < 3/2$; the relative benefits of the better technology are not so great. Errors are no longer required to leave C_2. If two adjacent A players form a coalition, they can gain by switching to B and achieving a payoff of 3, which is higher than the payoff of 2α they attain in state C_2. In the absence of random errors, C_2 collapses and the process returns to $B^{|N|}$. More than two errors are required to exit the basin of attraction of $B^{|N|}$. Following three errors on a diagonal, the process can attain the state C_3 which has a 3 by 3 block of players playing A. This block of players playing A can expand with the aid of a single error. It is not possible to leave C_3 without the help of errors, no matter how close α is to 1: the players who play A in C_3 form what will later be formally defined as a *parochial* set. To see that errors are required to leave C_3, first consider the player in the center of the square. He attains his maximum possible payoff of 4α, so he will not intentionally change his action as part of a coalition or otherwise. Secondly, consider the neighbors of the central player. Their payoffs at C_3 are 3α, so they will never intentionally change their action unless the central player also changes his, which he will not. The players at the corners of the block of A players cannot earn more than their C_3 payoff of 2α unless some non-corner player in the square changes his action, which will not occur. Similar arguments to those in the case $k = 1$ lead us to conclude that $W_{4,\alpha,\varepsilon}(B^{|N|}, A^{|N|}) \in \Theta(\varepsilon^{-3})$. Convergence to the efficient social norm is an order of magnitude slower in the presence of coalitional behavior: small outbreaks of innovation are snuffed out as the players involved in the outbreak collaborate to recoordinate with the population as a whole. The possibility of coalitional behavior has led to a *conservative* effect.

Now, consider the case $3/2 \le \alpha < 2$, $k = 4$. From state C_2, pairs of players who play strategy B in C_2, and who each have a neighbor playing A, can switch together to playing A and attaining a payoff of 2α which is greater than their payoff of 3 in C_2. In this way, the set of players playing A can expand without errors. This speeds up the process of moving to the efficient norm. However, to reach C_2 the initial two errors are still necessary so the order of magnitude of the wait is the same as that for the case without coalitional behavior, $W_{4,\alpha,\varepsilon}(B^{|N|}, A^{|N|}) \in \Theta(\varepsilon^{-2})$.

Finally, consider the case $2 \le \alpha$, $k = 4$. From state $B^{|N|}$, a square of 4 players can form a coalition. By switching to playing A, thus reaching state C_2, they can attain a payoff of 2α which is greater than their payoff at $B^{|N|}$ of 4. This can happen for any such set of players on the grid.[5] Therefore, the process can move to $A^{|N|}$ without the aid of any errors. $W_{4,\alpha,\varepsilon}(B^{|N|}, A^{|N|}) \in \Theta(1)$. Convergence to the efficient social norm is orders of magnitude faster in the presence of coalitional behavior: coalitions coordinate innovation in the population. The possibility of coalitional behavior has led to a *reforming* effect.

The reasoning of the preceding three paragraphs leads to the following proposition.

Proposition 1. *Let* **g** *be the* n_1 *by* n_2, $n_1 n_2 = |N|$, *square lattice on a torus with von Neumann neighborhoods,* $4 \le k \ll n_1, n_2$. *Then, as* $\varepsilon \to 0$,

$$\alpha < \frac{3}{2} \implies \frac{W_{k,\alpha,\varepsilon}(B^{|N|}, A^{|N|})}{W_{1,\alpha,\varepsilon}(B^{|N|}, A^{|N|})} \to \infty$$

$$\frac{3}{2} \le \alpha < 2 \implies \frac{W_{k,\alpha,\varepsilon}(B^{|N|}, A^{|N|})}{W_{1,\alpha,\varepsilon}(B^{|N|}, A^{|N|})} \in \Theta(1)$$

$$2 \le \alpha \implies \frac{W_{k,\alpha,\varepsilon}(B^{|N|}, A^{|N|})}{W_{1,\alpha,\varepsilon}(B^{|N|}, A^{|N|})} \to 0$$

Proposition 1 tells us that coalitional behavior can have either a *conservative* or a *reforming* effect. For low values of α, it has a conservative effect: when small groups of players start to play A and form a configuration which is stable under an individual best response dynamic, coalitions of players tear apart the cluster of deviant behavior, taking the process back to the state in which B is played by all. For large values of α, coalitional behavior has a reforming effect: groups of players can coordinate their choice to play A, increasing their payoffs from the change by

[5] Note that although the underlying game is a potential game with a potential function given by the sum of the payoffs of all the players, local maxima of the potential function are not necessarily absorbing states of the unperturbed dynamic when $k > 1$. For $\alpha < 3$, $B^{|N|}$ is a local maximum of the potential function. The move from $B^{|N|}$ to C_2 changes the potential by $8\alpha - 24$, which is negative for $\alpha < 3$, and yet the move from $B^{|N|}$ to C_2 occurs under the unperturbed dynamic for $k = 4$ and $\alpha \ge 2$.

ensuring that it occurs simultaneously to that of their neighbors. This speeds up the process of convergence to the efficient social norm.[6]

Note that $W_{k,\alpha,\varepsilon}(B^{|N|}, A^{|N|})$ is not necessarily monotonic in k. If $k = |N|$, there is the possibility of the coalition $S = N$ being chosen to respond, and the players in N choosing to play $A^{|N|}$ in a single step without the aid of any errors. Convergence is fast and the formation of the grand coalition has a reforming effect.

The question arises as to how far the results of the preceding paragraphs can be extended to general network architectures. The answer is sometimes unambiguously positive and sometimes not. It is simple to show that there always exists a reforming effect for large enough α or for $k = |N|$ (see Newton and Angus [18] for details). Conservative effects are more nuanced. To analyze them, we first need some definitions pertaining to the process with $\varepsilon = 0$.

4. Deterministic contagion

The *basin of attraction* of a stable state $x \in \Lambda_{k,\alpha}$ is the set of states from which, in the absence of errors, convergence to x is guaranteed. The *basin of possible attraction* is the set of states from which, in the absence of errors, convergence to x is possible. Allowing coalitions of a greater size reduces the set of stable states. This is equivalent to the fact that for any game, the set of $(k+1)$-strong Nash equilibria is a subset of the set of k-strong Nash equilibria. Furthermore, an expansion of the set of allowable coalitions also (weakly) expands the basins of possible attraction of stable states, as any path which was possible with a lower k, is also possible with a higher k. No such monotonicity exists for basins of attraction.[7]

Definition 3. The basin of attraction of $x \in \Lambda_{k,\alpha}$ is defined as:

$$D_{k,\alpha}(x) = \left\{ y \in X : P^t_{k,\alpha,0}(y,x) \to 1 \text{ as } t \to \infty \right\}$$

Definition 4. The basin of possible attraction of $x \in \Lambda_{k,\alpha}$ is defined as:

$$\bar{D}_{k,\alpha}(x) = \left\{ y \in X : \sum_{t=1}^{\infty} P^t_{k,\alpha,0}(y,x) > 0 \right\}$$

Proposition 2. *Assume $k_1 \le k_2$. Then $\Lambda_{k_1,\alpha} \supseteq \Lambda_{k_2,\alpha}$. Moreover, $x \in \Lambda_{k_2,\alpha}$ implies that $\bar{D}_{k_1,\alpha}(x) \subseteq \bar{D}_{k_2,\alpha}(x)$.*

As an interesting point of comparison with the conservative effects observed in our setup, we can consider the effect of coalitional behavior on deterministic 'contagion' in the style of

[6] Montanari and Saberi [13] would consider all of these parameter specifications to give fast convergence as the order of magnitude of the waiting time does not increase in population size. Given that period length is undefined, for fixed small ε this could encompass massive differences in actual waiting times (see simulations in Newton and Angus [18]). The focus of the current paper is not whether convergence is 'fast' or 'slow' as such, but on the effects of coalitional behavior relative to the baseline process without coalitional behavior.

[7] By increasing the size of basins of possible attraction, an increase in k will decrease the size of basins of attraction, *as long as the set of stable states remains the same*. If the set of stable states changes, this is no longer the case. Consider the example of Section 3 with $\alpha < 3/2$. $C_2 \notin D_{1,\alpha}(B^{|N|})$, but for $k = 4$, C_2 is no longer stable and $C_2 \in D_{4,\alpha}(B^{|N|})$.

Morris [14]. For any k, we characterize the set of states from which the spread of A across the entire population under the unperturbed ($\varepsilon = 0$) dynamic can occur.[8]

Proposition 3. *For any $x \in X$, $x \in \bar{D}_{k,\alpha}(A^{|N|})$ if and only if there does not exist $S \subseteq \{i \in N : x_i = B\}$ such that:*

$$\forall T \subseteq S, |T| \leq k, \exists i \in T : \quad \frac{|N_i \setminus S| + |N_i \cap T|}{|N_i| + |N_i \cap T|} < \frac{1}{1+\alpha}$$

That is, contagion cannot occur if there exists a set of players which (i) is insular enough to protect it from being 'infected' by A by the rest of the population but (ii) does not contain subsets of players which are themselves insular enough to coordinate their switch to A. When $k = 1$, T must be a singleton, so $N_i \cap T$ is empty and we have, in essence, the result of Morris [14]. The proof, however, is more similar to that of Easley and Kleinberg [3]. Note that Proposition 2 implies that the larger is k, the larger is the basin of possible attraction of $A^{|N|}$, from which contagion can occur. However, the size of the basin of possible attraction is only part of the story, and we have already seen in Section 3 that although larger k increases the size of $\bar{D}_{k,\alpha}(A^{|N|})$, it can also increase the waiting time until it is reached. To emphasize: increasing k has a monotonic effect on deterministic contagion in the style of Morris [14]. As we have seen in Section 3, it can have a non-monotonic effect on the contagion of the current model.

5. Conservative effects

As noted at the end of Section 3, a reforming effect of coalitional behavior is always possible for large enough values of α. Is a conservative effect similarly always possible? The answer is no, as can be seen if we consider ring networks. In such a network the vertices are arranged in a circle and each vertex is connected to m neighbors on either side. For $k = 1$, the only absorbing states of the unperturbed dynamic are $A^{|N|}$ and $B^{|N|}$. No intermediate stable states exist for any m, α. The only possible effect of coalitional behavior is then to speed the transition. This is a general result in the absence of intermediate equilibria. From any state, without random errors, the process will converge to either $A^{|N|}$ or $B^{|N|}$. Any number of errors that is enough to move the process into the basin of possible attraction of $A^{|N|}$ when $k = 1$ is also enough to move the process into the basin of possible attraction of $A^{|N|}$ when $k > 1$.

Proposition 4. *If for $k = 1$, $\Lambda_{k,\alpha} = \{A^{|N|}, B^{|N|}\}$, then for any $k \geq 1$,*

$$\frac{W_{k,\alpha,\varepsilon}(B^{|N|}, A^{|N|})}{W_{1,\alpha,\varepsilon}(B^{|N|}, A^{|N|})} \in O(1)$$

So, rings and square lattices give very different results when it comes to predicting the impact of coalitional behavior on adaptive dynamics. This is important as each is a commonly used model of local interaction. Another important network that satisfies the conditions of Proposition 4 is the complete network, in which every player neighbors every other player. The complete network and the ring are very different networks: in the class of connected networks with $|N|$

[8] Morris [14] pays specific attention to countably infinite populations for which the basin of possible attraction contains states in which a finite number of players play A.

vertices, the complete network has the greatest number of edges ($\frac{|N|(|N|-1)}{2}$); the ring with $m = 1$ has $|N|$ edges, one more than the lowest possible number.

Now we turn our attention to a method for showing the existence of a conservative effect for small enough α. This requires us to define the notion of a *parochial* set of players.

Definition 5. For $S \subseteq N$, define:

$$I_0(S) = \{i \in S : N_i \subseteq S\},$$
$$I_m(S) = \big\{i \in S : |N_i \setminus S| \leq \big|N_i \cap I_{m-1}(S)\big|\big\}, \quad m \geq 1.$$

Note that $I_{m-1}(S) \subseteq I_m(S)$. We say that S is *parochial* if there exists $m \geq 0$ such that $S = I_m(S)$.

Note that the definition of a parochial set does not depend on the value of α. A parochial set, S, always contains a set of players, $I_0(S)$, who are completely isolated from the network outside of S. $I_1(S)$ is then formed from all of the players in S who have at least as much exposure to $I_0(S)$ as they have to the network outside of S. $I_2(S)$ is similarly defined, and so on. Intuitively, the recursive definition means that every member of a parochial set has at least as many neighbors who are more deeply embedded in the set than he is, than he has neighbors outside of the set. Every member of a parochial set will have at least half of his neighbors within the set, but this fact alone does not suffice to make a set parochial. In fact, any set of players, S, in which every player has at least one neighbor outside of the set cannot be parochial, as $I_0(S)$ will be empty.

Define $\mathcal{P}_A$ as the set of states such that the set of players playing A contains a parochial subset.

Definition 6.

$$\mathcal{P}_A = \big\{x \in X : \exists S \subseteq \{i \in N : x_i = A\} \text{ such that } S \text{ is parochial}\big\}.$$

If S is the set of players playing A and S is not parochial then there exist nonempty sets of A players who are not in $I_m(S)$ for any m, and who, for some k and α, can gain by coordinating a switch back to B. Iterating, the process can return to either $B^{|N|}$, or a state in which the set of players playing A is a parochial set. Conversely, if some parochial set of players is playing A, then for any values of k and α, no member of the set will ever switch to B unless at least one member of the set makes an error. The reason for this is that when a parochial set S is playing A, any player in $I_0(S)$ is earning his maximum possible payoff and so will not participate in any coalitional deviation. Now, fixing players in $I_0(S)$ to play A, there is no way that any player in $I_1(S)$ can switch to B and earn a payoff at least as high as his current payoff, regardless of the actions of other players outside of $I_0(S)$. Iterating this logic, no player in S will participate in a coalitional deviation to play B.

Proposition 5. *$x \in \mathcal{P}_A$ if and only if $\nexists k, \alpha$ such that $x \in \bar{D}_{k,\alpha}(B^{|N|})$.*

So, for every $x \notin \mathcal{P}_A$, there exist k_x and α_x such that $x \in \bar{D}_{k_x,\alpha_x}(B^{|N|})$. $\bar{D}_{k,\alpha}(B^{|N|})$ is monotonic in k and α, so choosing k^* to equal the largest k_x and α^* to equal the smallest α_x, we have that $x \notin \mathcal{P}_A$ implies $x \in \bar{D}_{k^*,\alpha^*}(B^{|N|})$. As the process is time homogeneous and has a finite state space, $\Phi_{k^*,\alpha^*,0}$ will either enter $\mathcal{P}_A$ or reach state $B^{|N|}$. That is, any intermediate stable state must be in $\mathcal{P}_A$. This bounds the waiting time to reach $A^{|N|}$ from below by the waiting time to reach $\mathcal{P}_A$. That is, the waiting time to reach $A^{|N|}$ is at least of the order of ε to the power of the number of errors required to reach a state in which some parochial subset of players play A.

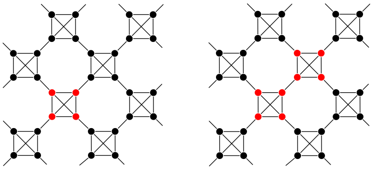

Fig. 3. Interconnected cliques.

The use of this result can be seen in that for the square lattice, state C_3 is in the set $\mathcal{P}_A$. Moreover, C_3 is among the elements of $\mathcal{P}_A$ which require the fewest errors to attain. For α close to 1, three errors are required to reach C_3 from $B^{|N|}$. From the above discussion, we know we can choose $k = k^*$ (in this case, $k = 4$ suffices) such that there are no intermediate stable states outside of $\mathcal{P}_A$. So, the waiting time to reach $A^{|N|}$ is at least of the order of the waiting time to reach $\mathcal{P}_A$, that is $\Theta(\varepsilon^{-3})$. As the waiting time to reach $A^{|N|}$ without coalitional behavior is $O(\varepsilon^{-2})$, this implies the existence of a conservative effect.

The same cannot be said for the regular network of connected cliques (Fig. 3), which illustrates that the failure of the condition in Proposition 4, that is to say the existence of intermediate stable states when $k = 1$, is necessary but not sufficient for the existence of a conservative effect. Assume α is close to 1. Consider $k = 4$. Although it is true that $Cli_2 \in \Lambda_{1,\alpha}$ and $Cli_2 \notin \Lambda_{k=4,\alpha}$, there is no conservative effect. The reason for this is that when $k = 1$, two errors are required to reach Cli_2. When $k = 4$, only a single error in which a player plays A is required, following which the three other members of the clique can better respond by switching to A. However, as $Cli_2 \notin \Lambda_{k=4,\alpha}$, we have still not reached an intermediate stable state. We require one further error (making a total of two) to move the process to $Cli_3 \in \mathcal{P}_A$. From Cli_3, the process can expand via single error driven steps between states in $\Lambda_{k=4,\alpha}$ until $A^{|N|}$ is reached. Therefore the waiting time for $k = 1$ and $k = 4$ is $\Theta(\varepsilon^{-2})$.

6. Conclusion

In settings in which every member of a population has a common interest in coordinating on a given efficient action, it might be expected that overt cooperation by coalitions of players in their choice of action would facilitate the spread of that action in the population. This paper has shown that this is not always the case and that there can also exist conservative effects by which coalitions slow the spread of efficient behavior in a population. Further examples, results for specific networks, and simulations illustrating the breadth and importance of these phenomena can be found in Newton and Angus [18]. Implications for the study of social dynamics and network design clearly exist and merit subsequent study.

Acknowledgments

The authors would like to thank Murali Agastya, Suren Basov, Larry Blume, Zhiwei Cui, David Easley, Sung-Ha Hwang, Claudio Mezzetti, Birendra Rai, Ryoji Sawa, Russell Toth and Galina Zudenkova for detailed comments and fruitful questions, as well as seminar audiences at UNSW, University of Melbourne, Boston University, Cornell University, McGill University, University of Lausanne, University of Zurich, Seoul National University, Sogang University and ANU.

Appendix A. Proofs

We use the concepts of and similar notation to Ellison [5].[9] For $x, y \in X$, define the resistance $r(x, y)$ so that the most probable transition from x to y occurs with probability of order $\varepsilon^{r(x,y)}$.

$$r(x, y) = \min\left\{r \in \mathbb{R}_+ : \lim_{\varepsilon \to 0} \frac{P_{k,\alpha,\varepsilon}(x, y)}{\varepsilon^r} > 0\right\} \wedge \infty.$$

For $Y \subseteq X$, let $\mathcal{S}(x, Y)$ be the set of all paths (sequences of distinct states) $\{x^1, \dots, x^T\}$ such that $x^1 = x$, $x^T \in Y$, $r(x^t, x^{t+1}) < \infty$ for $t = 1, \dots, T - 1$. Define

$$r\left(x^1, \dots, x^T\right) = \sum_{t=1}^{t=T-1} r\left(x^t, x^{t+1}\right); \qquad r(x, Y) = \min_{\{x^t\}_{t=1}^T \in \mathcal{S}(x,Y)} r\left(x^1, \dots x^T\right).$$

Then, the *radius* of $x \in \Lambda_{k,\alpha}$ is

$$R(x) = r\left(x, X \setminus D_{k,\alpha}(x)\right).$$

The radius is the resistance of the lowest resistance path from x to outside the basin of attraction of x. For $x \notin \Lambda_{k,\alpha}$, set $R(x) = 0$. For a path $\{x^1, \dots, x^T\}$, let $\{\bar{x}^1, \dots, \bar{x}^\tau\}$ be the sequence of states in $\Lambda_{k,\alpha}$ through which the path passes consecutively. Define modified resistance[10]:

$$r^*\left(x^1, \dots, x^T\right) = r\left(x^1, \dots, x^T\right) - \sum_{t=2}^{t=\tau-1} R\left(\bar{x}^t\right)$$

and

$$r^*(x, Y) = \min_{\{x^t\}_{t=1}^T \in \mathcal{S}(x,Y)} r^*\left(x^1, \dots x^T\right).$$

Finally, define the *modified coradius* of $y \in \Lambda_{k,\alpha}$ as

$$CR^*(y) = \max_{x \neq y} r^*\left(x, \{y\}\right).$$

For purposes of comparison, these quantities will sometimes be given subscripts k, α.

Proof of Proposition 1. For $k = 1$, $\alpha \geq 3$, a single error (and no fewer) is enough to move the process to $A^{|N|}$. For $\alpha < 3$, two errors move the process to C_2. No fewer than two errors suffice to move the process out of $D_{k,\alpha}(B^{|N|})$. From C_2, a single error then suffices to move the process to C_3, and so on. Noting that from any state not equal to $B^{|N|}$, a single error is enough to give an expanding set of squares, and that $R_{1,\alpha}(C_i) = 1$, we have

$$\alpha < 3 \implies CR^*_{1,\alpha}\left(A^{|N|}\right) = 2; \qquad \alpha \geq 3 \implies CR^*_{1,\alpha}\left(A^{|N|}\right) = 1.$$

[9] Ellison [5] cites a no longer extant working paper of Evans as containing the first statements and use of some of these concepts.

[10] The reason that modified resistances and associated quantities are useful is that when using spanning tree arguments as found in Freidlin and Wentzell [6], when edges are added to an existing tree, other edges must be deleted for the graph to remain a tree. Theorems using radii and coradii to give sufficient conditions for stochastic stability follow trivially from this observation.

For $4 \leq k \ll n_1, n_2$, for $\alpha < 3/2$, two errors do not suffice to leave $D_{k,\alpha}(B^{|N|})$. Three errors, however, can take the process to C_3, from where a single error can take the process to C_4 and so on. Note that $R_{k,\alpha}(C_i) = 1$ for $i \geq 3$. For $3/2 \leq \alpha < 2$, two errors are required to move to $C_2 \in \bar{D}_{k,\alpha}(A^{|N|})$. For $\alpha \geq 2$, $B^{|N|} \in D_{k,\alpha}(A^{|N|})$. We have

$$\alpha < 3/2 \implies CR^*_{k,\alpha}\big(A^{|N|}\big) = 3; \qquad \alpha \geq 2 \implies CR^*_{k,\alpha}\big(A^{|N|}\big) = 0;$$
$$3/2 \leq \alpha < 2 \implies CR^*_{k,\alpha}\big(A^{|N|}\big) = 2.$$

Note that in all of the above cases, $CR^*_{k,\alpha}(A^{|N|}) = R_{k,\alpha}(B^{|N|})$. The value of $R_{k,\alpha}(B^{|N|})$ gives a lower bound on $W_{k,\alpha,\varepsilon}(B^{|N|}, A^{|N|})$. The value of $CR^*_{k,\alpha}(A^{|N|})$ gives an upper bound on $W_{k,\alpha,\varepsilon}(B^{|N|}, A^{|N|})$ by Theorem 2 of [5]. □

Proof of Proposition 2. The set of feasible coalitions is independent of k, so $k_1 \leq k_2$ implies

$$\mathcal{N}(k_1) \subseteq \mathcal{N}(k_2) \implies \operatorname{supp}(F_{k_1}) \subseteq \operatorname{supp}(F_{k_2})$$

so for all $x, y \in X$,

$$P_{k_1,\alpha,0}(x, y) > 0 \implies P_{k_2,\alpha,0}(x, y) > 0$$

and

$$P_{k_1,\alpha,0}\big(x, X \setminus \{x\}\big) > 0 \implies P_{k_2,\alpha,0}\big(x, X \setminus \{x\}\big) > 0$$

which implies

$$x \notin \Lambda_{k_1,\alpha} \implies x \notin \Lambda_{k_2,\alpha}$$

which implies $\Lambda_{k_1,\alpha} \supseteq \Lambda_{k_2,\alpha}$. Furthermore, for $x \in \Lambda_{k_2,\alpha}$,

$$\begin{aligned} y \in \bar{D}_{k_1,\alpha}(x) &\implies P^t_{k_1,\alpha,0}(y, x) > 0 \quad \text{for some } t \in \mathbb{N}_+ \\ &\implies P^t_{k_2,\alpha,0}(y, x) > 0 \implies y \in \bar{D}_{k_2,\alpha}(x). \qquad \square \end{aligned}$$

Proof of Proposition 3. For given $x \in X$, assume there exists $S \subseteq \{i \in N : x_i = B\}$ such that

$$\forall T \subseteq S, |T| \leq k, \exists i \in T: \quad \frac{|N_i \setminus S| + |N_i \cap T|}{|N_i| + |N_i \cap T|} < \frac{1}{1+\alpha}.$$

Take such an S. Then for all $T \subseteq S$, $|T| \leq k$, there exists $i \in T$ such that

$$\alpha\big(|N_i \setminus S| + |N_i \cap T|\big) < |N_i \cap S| \leq u_i(x).$$

The left hand side is the maximum payoff attainable by player i from playing A if all players in $S \setminus T$ play B. So no subset of players in S of size $\leq k$ will switch to A unless some subset of players in S have already switched to A. Therefore $x \notin \bar{D}_{k,\alpha}(A^{|N|})$. This proves the 'only if' part of the proposition.

To prove the 'if' part of the proposition assume that $x \notin \bar{D}_{k,\alpha}(A^{|N|})$. Starting from state $x = x^1$, if there is any feasible coalition $U \subseteq N$, $|U| \leq k$, $x_U \neq A^{|U|}$, such that for all $i \in U$, $u_i(x_U = A^{|U|}, x^t_{-U}) \geq u_i(x^t)$, then with some probability U better responds and the state moves to $x^{t+1}_U = A^{|U|}$, $x^{t+1}_{-U} = x^t_{-U}$. Iterate until there is no such subset of players, say at time τ. Let $S = \{i \in N : x^\tau_i = B\}$. This set must be nonempty or else $x^\tau = A^{|N|}$, which would contradict

$x \notin \bar{D}_{k,\alpha}(A^{|N|})$. Now, for all $T \subseteq S$, $|T| \leq k$, as at least one player, say player i, in T would be strictly worse off if T switched to action A, we have

$$\alpha\big(|N_i \setminus S| + |N_i \cap T|\big) < u_i\big(x^\tau\big) = |N_i \cap S| = |N_i| - |N_i \setminus S|.$$

Rearranging gives

$$\frac{|N_i \setminus S| + |N_i \cap T|}{|N_i| + |N_i \cap T|} < \frac{1}{1+\alpha}.$$

and we have our result. □

Proof of Proposition 4. $\Lambda_{1,\alpha} = \{B^{|N|}, A^{|N|}\}$. Note that, by Proposition 2, $\bar{D}_{1,\alpha}(A^{|N|}) \subseteq \bar{D}_{k,\alpha}(A^{|N|})$ for all $k \geq 1$. So

$$\begin{aligned} CR^*_{1,\alpha}\big(A^{|N|}\big) &= R_{1,\alpha}\big(B^{|N|}\big) = r_{1,\alpha}\big(B^{|N|}, \bar{D}_{1,\alpha}\big(A^{|N|}\big)\big) \\ &\geq r_{1,\alpha}\big(B^{|N|}, \bar{D}_{k>1,\alpha}\big(A^{|N|}\big)\big) \geq r_{k>1,\alpha}\big(B^{|N|}, \bar{D}_{k>1,\alpha}\big(A^{|N|}\big)\big) \\ &= R_{k>1,\alpha}\big(B^{|N|}\big) = CR^*_{k>1,\alpha}\big(A^{|N|}\big) \qquad \square \end{aligned}$$

Proof of Proposition 5. First the 'only if' part of the statement is addressed. Let $x \in \mathcal{P}_A$ and $S \subseteq \{i \in N: x_i = A\}$ be parochial. $I_0(S)$ must be nonempty. For any α, $i \in I_0(S)$, $N \supseteq T \supseteq \{i\}$, as

$$u_i(x) = \alpha|N_i| > |N_i| \geq u_i(\tilde{x}_i = B, \hat{x}_{-i}) \quad \text{for any } \hat{x}_{-i},$$

we have

$$\tilde{x}_T \in A_T(x) \quad \Longrightarrow \quad \tilde{x}_i = A.$$

Then, by induction, for any $i \in I_m(S)$, $m \geq 1$, $N \supseteq T \supseteq \{i\}$, as

$$\begin{aligned} u_i(x) &\geq \alpha\big(|N_i| - |N_i \setminus S|\big) > |N_i| - |N_i \setminus S| \geq |N_i| - \big|N_i \cap I_{m-1}(S)\big| \\ &\geq u_i\big(\tilde{x}_i = B, \tilde{x}_{I_{m-1}(S)} = A^{|I_{m-1}(S)|}, \hat{x}_{-(I_{m-1}(S)\cup\{i\})}\big) \quad \text{for any } \hat{x}_{-(I_{m-1}(S)\cup\{i\})}, \end{aligned}$$

we have

$$\tilde{x}_T \in A_T(x) \quad \Longrightarrow \quad \tilde{x}_i = A.$$

So, for any $x^t \in \mathcal{P}_A$, $x_S^{t+1} = A^{|S|}$ and therefore $x^{t+1} \in \mathcal{P}_A \not\ni B^{|N|}$. So $x \in \mathcal{P}_A$ implies $x \notin \bar{D}_{k,\alpha}(B^{|N|})$.

Now the 'if' part of the statement is addressed. Assume that $x \notin \bar{D}_{k,\alpha}(B^{|N|})$ for any k, α. If there exists feasible $S \subseteq N$ such that

$$\begin{aligned} & x_S \neq B^{|S|} \quad \text{and} \\ & \exists \underline{\alpha}: \forall i \in S, \quad u_i\big(B^{|S|}, x_{-S}\big) \geq u_i(x), \end{aligned} \tag{$*$}$$

then, assuming $k \geq |S|$, we have $S \in \text{supp}(F_k)$. Assuming $\alpha < \underline{\alpha}$,

$$B^{|S|} \in A_S(x) \quad \text{so } P_{k,\alpha,0}\big(x, \big(B^{|S|}, x_{-S}\big)\big) > 0.$$

Starting from $x^t \notin \bar{D}_{k,\alpha}(B^{|N|})$, let $x^{t+1} = (B^{|S|}, x_{-S})$ for such an S, and iterate until a state, $\tilde{x}$, is reached such that there does not exist feasible S which satisfies $(*)$. Let $T = \{i \in N: \tilde{x}_i = A\}$. Then $I_0(T) \neq \emptyset$ or some feasible subset of T would satisfy $(*)$ as there would exist α such that

$$\forall i \in T, \quad u_i\big(B^{|T|}, \tilde{x}_{-T}\big) = |N_i| \geq \alpha\big(|N_i| - 1\big) \geq u_i(\tilde{x}).$$

If $\nexists m$ such that $I_m(T) = T$, then choose m such that $I_m(T) = I_{m-1}(T)$. Then

$$\forall i \in T \setminus I_m(T), \quad |N_i \setminus T| > \big|N_i \cap I_{m-1}(T)\big| = \big|N_i \cap I_m(T)\big|.$$

But then, for all $i \in T \setminus I_m(T)$, there exists α such that

$$\begin{aligned} u_i\big(B^{|T \setminus I_m(T)|}, \tilde{x}_{-(T \setminus I_m(T))}\big) &\geq |N_i \setminus T| + \big|N_i \cap \big(T \setminus I_m(T)\big)\big| \\ &> \alpha\big(\big|N_i \cap I_m(T)\big| + \big|N_i \cap \big(T \setminus I_m(T)\big)\big|\big) = \alpha|N_i \cap T| \\ &\geq u_i(\tilde{x}) \end{aligned}$$

so some feasible subset of $T \setminus I_m(T)$ satisfies $(*)$ and we have a contradiction. Therefore $\exists m$ such that $I_m(T) = T$. T is a parochial set such that $x_T = A^{|T|}$. Therefore $x \in \mathcal{P}_A$. □

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