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Bounding the cop number of a graph by its genus. (English summary)

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Cops and robbers is a game played on a graph $G = (V, E)$ by two players. One player controls a set of $k \geq 1$ *cops* and the other player controls a *robber*. Initially the first player chooses a starting configuration $(c_1, \ldots, c_k) \in V^k$ for the cops, following which the second player chooses a starting vertex $r \in V$ for the robber. In each round, the cops player may leave any given cop where it is on the graph or move it to an adjacent vertex. Following this, the robber player may leave the robber where it is or move it to an adjacent vertex. The cops player wins if at some round there is a cop on the same vertex as the robber. Otherwise the robber wins. The *cop number* $c(G)$ of a graph G is the smallest k such that the cops player has a strategy that guarantees that he will win the game.

The cop number can be bounded in terms of the *genus* of the graph. The genus of G is the smallest m such that G can be drawn on an orientable surface of genus m without any crossings. Such bounds have been proven using the result of M. S. Aigner and M. Fromme [Discrete Appl. Math. **8** (1984), no. 1, 1–11; MR0739593] that the shortest path between any two vertices is guardable. Specifically, a cop *guards* a set C of vertices if whenever the robber moves to a vertex in C she is caught in the next round. C is *guardable* if there is a strategy for a single cop that guards C after finitely many periods. By considering shortest paths between vertices and guarding these, the orientable surface on which G is drawn can be segregated into orientable surfaces of strictly lower genus. In terms of the game, the robber can be corralled into a subgraph of G that has strictly lower genus than G. Eventually the robber will be restricted to a planar subgraph of G on which three cops are required to catch the robber.

The authors improve upon the previous bounds in [B. S. W. Schröder, in *Categorical perspectives (Kent, OH, 1998)*, 243–263, Trends Math., Birkhäuser Boston, Boston, MA, 2001; MR1827672] by considering a *topological marker-cutter* game in which one player marks points on a surface and the other cuts the surface. This is related to the idea above of corralling the robber into smaller and smaller areas. For the details of the technique under discussion, the reader is referred to the paper's conference version [arXiv:1911.01758], by the same authors. *Jonathan Newton*