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Cops and robbers on graphs of bounded diameter. (English summary)

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Cops and robbers is a game played on a graph $G = (V, E)$ by two players. One player controls a set of $k \geq 1$ *cops* and the other player controls a *robber*. Initially the first player chooses a starting configuration $(c_1, \ldots, c_k) \in V^k$ for the cops, following which the second player chooses a starting vertex $r \in V$ for the robber. In each round, the cops player may leave any given cop where it is on the graph or move it to an adjacent vertex. Following this, the robber player may leave the robber where it is or move it to an adjacent vertex. The cops player wins if at some round there is a cop on the same vertex as the robber. Otherwise the robber wins. The *cop number* $c(G)$ of a graph G is the smallest k such that the cops player has a strategy that guarantees that he will win the game.

A. D. Scott and B. Sudakov [SIAM J. Discrete Math. **25** (2011), no. 3, 1438–1442; MR2837608] showed that the cop number of any connected n-vertex graph of diameter d is at most n^t, where $t = 1 - \frac{1}{\lceil \log d \rceil + 1} + o(1)$ and the logarithm is base 2. The main theorem of the paper under discussion (Theorem 8) improves this bound to $t = 1 - \frac{2}{2\lceil \log d \rceil + 1} + o(1)$.

The proof strategy involves the consideration of randomly placed cops. It is shown that for any given set of vertices A (we can think of these as where the robber might be), if we consider any large enough ball of vertices around A, then, for suitably chosen probabilities, these balls will usually contain more randomly placed cops than there are vertices within A. Subsequently, arguments are given to show how such sets of cops can ensure that any robber that starts within A is eventually caught. This style of argument is iterated to get finer bounds on the cop number than are possible with a single iteration of the argument. *Jonathan Newton*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.