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Dynamic matching pennies on networks. (English summary)

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This paper considers undirected networks in which the vertex set N represents *players*. Each player has two possible actions, 0 and 1. There are two *types* of players: (i) *conformists* who obtain a payoff of 1 from every neighbor (on the network) who plays the same action as they do and obtain a payoff of -1 from every neighbor who plays the other action, and (ii) *rebels* who obtain a payoff of -1 from every neighbor who plays the same action as they do and obtain a payoff of 1 from every neighbor who plays the other action.

A discrete time Markov process on the state space $N^{\{0,1\}}$ is defined. Every period, each player who currently obtains a strictly negative total payoff changes his action. This defines the (synchronous) *best response dynamic*.

The paper studies the length of (i.e., number of states in) absorbing cycles of this process. The initial results consider the case in which all players are of the same type (Theorem 1) and the case in which every player has no neighbor of the same type and there are no two neighboring players who each have an even number of neighbors (Theorem 2). The later results consider three specific networks: the line, the ring and the star.

Theorem 1 gives cycle lengths of 1 and 2 depending on the initial state. This is similar to the two-player game ($|N|=2$) between two conformists (that is, a coordination game).

Theorem 2 gives a cycle length of 4. This is similar to the two-player game ($|N|=2$) between one conformist and one rebel (that is, a matching pennies game).

Both Theorems 1 and 2 are constructed by finding an appropriate Lyapunov function. Consideration of the Lyapunov function then gives a relationship between actions taken at times $t-1$ and $t+1$, conditional on the process being within an absorbing cycle. For example, in the case of Theorem 2, it implies that any player must play different actions at times $t-1$ and $t+1$. That is, within an absorbing cycle, a player's sequence of actions must be $\ldots 0x1y0x1y0\ldots$ with $x\neq y$. For either $x=0$ or $x=1$, this gives $\ldots 00110011\ldots$. That is, the cycle has length 4.

The results for specific networks find that in almost all cases cycle lengths are 1, 2 or 4. This is because, whenever there exist adjacent players of the same type, the regularity imposed by their coordination or anti-coordination imposes regularity on their neighbors, thus on their neighbors' neighbors and so on. When there exist no pairs of adjacent players of the same type, the line and star network are analogous to two-player matching pennies and we obtain a cycle length of 4. This is similar to Theorem 2 but with adjacent players of even degree. In contrast, when there exist no pairs of adjacent players of the same type, the ring network admits cycle lengths that grow linearly in the number of players. *Jonathan Newton*

References

1. Anderlini L, Ianni A (1996) Path dependence and learning from neighbors. Games Econ Behav 13(2):141–177 MR1391300
2. Apt KR, de Keijzer B, Rahn M, Schäfer G, Simon S (2017) Coordination games on graphs. Int J Game Theory 46(3):851–877 MR3674216
3. Arthur WB (1994) Inductive reasoning and bounded rationality. Am Econ Rev 84(2):406–411
4. Balister P, Bollobás B, Johnson JR, Walters M (2010) Random majority percolation. Random Struct Algorithms 36(3):315–340 MR2650054
5. Berninghaus SK, Schwalbe U (1996) Conventions, local interaction, and automata networks. J Evol Econ 6(3):297–312
6. Bramoullé Y (2007) Anti-coordination and social interactions. Games Econ Behav 58(1):30–49 MR2285215
7. Bramoullé Y, Kranton R (2016) Games played on networks. In: Bramoullé Y, Galeotti A, Rogers B, Rogers BW (eds) The Oxford handbook of the economics of networks, chapter 5. Oxford University Press, Oxford, pp 83–112
8. Bramoullé Y, López-Pintado D, Goyal S, Vega-Redondo F (2004) Network formation and anti–coordination games. Int J Game Theory 33(1):1–19 MR2109594
9. Bramoullé Y, Kranton R, D'amours M (2014) Strategic interaction and networks. Am Econ Rev 104(3):898–930
10. Cannings C (2009) The majority and minority models on regular and random graphs. In: International conference on GameNets' 09. IEEE, New York, pp 1–16
11. Cao Z, Gao H, Qu X, Yang M, Yang X (2013) Fashion, cooperation, and social interactions. PLoS ONE 8(1):e49441
12. Cao Z, Yang X (2014) The fashion game: network extension of matching pennies. Theor Comput Sci 540:169–181 MR3215032
13. Challet D, Zhang Y-C (1997) Emergence of cooperation and organization in an evolutionary game. Phys A Stat Mech Appl 246(3):407–418
14. Chen H-C, Chow Y, Wu L-C (2013) Imitation, local interaction, and coordination. Int J Game Theory 42(4):1041–1057 MR3111673
15. Chiappori P-A, Levitt S, Groseclose T (2002) Testing mixed-strategy equilibria when players are heterogeneous: the case of penalty kicks in soccer. Am Econ Rev 92(4):1138–1151
16. Easley D, Kleinberg J (2010) Networks, crowds, and markets: reasoning about a highly connected world. Cambridge University Press, Cambridge MR2677125
17. Ellison G (1993) Learning, local interaction, and coordination. Econometrica 61(5):1047–1071 MR1234793
18. Gärtner B, Zehmakan AN (2017) Color war: cellular automata with majority-rule. In: International conference on language and automata theory and applications. Springer, Berlin, pp 393–404 MR3639821
19. Goles E (1987) Lyapunov functions associated to automata networks. In: Centre National de Recherche Scientifique on Automata networks in computer science: theory and applications. Princeton University Press, Princeton, pp 58–81
20. Goyal S (2007) Connections: an introduction to the economics of networks. Princeton University Press, Princeton MR2356312
21. Granovetter M (1978) Threshold models of collective behavior. Am J Sociol 83(6):1420–1443
22. Haslegrave J, Cannings C (2017) Majority dynamics with one nonconformist. Discret Appl Math 219:32–39 MR3599455
23. Hernandez P, Muñoz-Herrera M, Sanchez A (2013) Heterogeneous network games: conflicting preferences. Games Econ Behav 79:56–66 MR3030914
24. Ilachinski A (2001) Cellular automata: a discrete universe. World Scientific, Singa-

pore MR1849342
25. Jackson MO (2008) Social and economic networks. Princeton University Press, Princeton MR2435744
26. Jackson MO, Zenou Y (2014) Games on networks. In: Peyton Y, Zamir S (eds) Handbook of game theory, chapter 3, vol 4. Elsevier Science, Amsterdam, pp 95–164
27. Kandori M, Mailath GJ, Rob R (1993) Learning, mutation, and long run equilibria in games. Econometrica 61(1):29–56 MR1201702
28. Kanoria Y, Montanari A (2011) Majority dynamics on trees and the dynamic cavity method. Ann Appl Probab 21(5):1694–1748 MR2884049
29. Menache I, Ozdaglar A (2011) Network games: theory, models, and dynamics. Morgan & Claypool Publishers, San Rafael
30. Monderer D, Shapley LS (1996) Potential games. Games Econ Behav 14(1):124–143 MR1393599
31. Palacios-Huerta I (2003) Professionals play minimax. Rev Econ Stud 70(2):395–415
32. Park A, Sabourian H (2011) Herding and contrarian behavior in financial markets. Econometrica 79(4):973–1026 MR2857879
33. Poljak S, Sura M (1983) On periodical behaviour in societies with symmetric influences. Combinatorica 3(1):119–121 MR0716427
34. Ramazi P, Riehl J, Cao M (2016) Networks of conforming or nonconforming individuals tend to reach satisfactory decisions. Proc Natl Acad Sci 113(46):12985–12990
35. Shirado H, Christakis NA (2017) Locally noisy autonomous agents improve global human coordination in network experiments. Nature 545(7654):18
36. Southwell R, Cannings C (2013) Best response games on regular graphs. Appl Math 4:950–962
37. Szabó G, Varga L, Borsos I (2014) Evolutionary matching-pennies game on bipartite regular networks. Phys Rev E 89(4):042820
38. Tamuz O, Tessler RJ (2015) Majority dynamics and the retention of information. Isr J Math 206(1):483–507 MR3319649
39. van Valen L (1980) Evolution as a zero-sum game for energy. Evol Theory 4:289–300
40. Young HP (1998) Individual strategy and social structure: an evolutionary theory of institutions. Princeton University Press, Princeton
41. Young HP (2001) The dynamics of conformity. In: Durlauf SN, Young HP (eds) Social dynamics. Brookings Institution Press, Washington, DC, pp 133–153
42. Zhang B, Cao Z, Qin C-Z, Yang X (2018) Fashion and homophily. Oper Res 66(6):1486–1497 MR3898841

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.