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Fashion game on graphs. (English summary)
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The paper considers a network version of matching pennies. Players are identified with the vertices of a simple graph. Each player can play one of two strategies. A strategy profile assigns a strategy to every player. There are two types of players. Given a strategy profile, the payoff of a *conformist* equals the number of his neighbors who play the same strategy as himself minus the number of his neighbors who play the alternative strategy. The payoff of a *rebel* equals the number of his neighbors who play a different strategy to himself minus the number of his neighbors who play the same strategy. A graph G is t-*satisfiable* if there exists a strategy profile such that every player obtains a payoff of at least t. The utility of G is the highest value of t for which G is t-satisfiable.

The paper determines the utility of paths, cycles and the complete graph. This is proved by case checking. It is also shown that any t-degenerate graph has a utility of at least $-t$. This is proven iteratively, at each stage removing a player with degree of at most t from the remaining graph (this is possible due to t-degeneracy) and choosing the player's strategy so that his payoff vis-à-vis players already removed is at least zero. His payoff vis-à-vis the players in the remaining graph cannot be less than $-t$ by definition.

The t-*satisfiability problem* is to determine whether a graph is t-satisfiable. It is shown that, for graphs containing both conformists and rebels, for $t \geq -2$ the t-satisfiability problem is NP-complete. This is proven by noting that 0-satisfiability is, in this context, equivalent to the existence of pure Nash equilibrium. It is shown that the problem of existence of a pure Nash equilibrium in a graph G is equivalent to determining t-satisfiability in some auxiliary graph. As the former problem is NP-complete [Z. G. Cao and X. G. Yang, Theoret. Comput. Sci. **540/541** (2014), 169–181; MR3215032], the latter problem must also be NP-complete.

It is further shown that, for graphs containing only rebels, for $t \geq 1$, the t-satisfiability problem is NP-complete. This is done by showing that the problem reduces to solving a specific type of graph coloring problem that is NP-complete. *Jonathan Newton*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.