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Finding a stable matching under type-specific minimum quotas. **(English summary)**
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This paper considers a discrete college-student matching problem. Each student can be matched to at most one college. Each college has a maximum number of students to which it can be matched. Each student has a preference ranking over colleges. Each college has a priority ranking over students. Each student has a type (e.g., yellow, green, blue, etc.). For each type, each college has a minimum and a maximum number of students of that type to which it can be matched. It is assumed that there exists a nonempty set of matchings that satisfies these type constraints. Attention is then restricted to this set.

A *fair* matching satisfies a notion of pairwise stability. Specifically, at a fair matching there do not exist students s, s', and college c' such that student s' is matched to college c' but (i) student s would rather be matched to c' than to his current college; (ii) college c' gives higher priority to s than it gives to s'; and (iii) it is possible to match s to c' and s' to some other college (leaving all other students at the same colleges) so that the type constraints are still satisfied.

A fair matching may fail to exist due to the following logic. Let there be two students s, s' of a given type, say red, both of which have college $\underline{c}$ as their least preferred college. Let there be one other student, say $\underline{s}$. Let college $\underline{c}$ have a minimum quota of one red student. Let there be two other colleges, c which gives highest priority to student s', then s, then $\underline{s}$, and c' which gives highest priority to student s, then s', then $\underline{s}$. Let c be the most preferred college of s and c' be the most preferred college of s'. Let every college have a capacity of at most one student.

One student must attend college $\underline{c}$. Without loss of generality, let this student be s. Then there are two possibilities for s'. Either s' attends his most preferred college c', in which case a pairwise deviation exists for s and c', following which s' can be matched to $\underline{c}$ so that the type constraints are not violated. Or s' attends college c (and $\underline{s}$ must attend college c'), in which case a pairwise deviation exists for s' and c', following which $\underline{s}$ attends college c and type constraints are not violated. Hence a fair matching does not exist.

This argument relies on c and c' having different priority rankings. If all colleges have a *common priority* ranking, for example, ranking s' ahead of s ahead of $\underline{s}$, then the lower-ranked red student, s, can be matched to $\underline{c}$, the higher-ranked red student, s', to his favourite college c', and $\underline{s}$ to c. Then there exists no pairwise deviation that does not lead to a violation of the type constraints. That is, the matching is fair.

The immediately preceding paragraph is essentially the main theorem of the paper (Theorem 2), which shows that a fair matching exists under a slight weakening of the common priority ranking assumption described above. *Jonathan Newton*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.