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Mutually best matches. (English summary)
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This paper gives an algorithm, *iterated formation of mutually best matches*, for allocating students to colleges in the *college admissions problem* of D. Gale and L. S. Shapley [Amer. Math. Monthly **69** (1962), no. 1, 9–15; MR1531503]. Each student has a strict preference ordering over colleges and each college has a strict preference ordering over students. It is possible that a student may find some colleges *unacceptable* (worse than remaining unmatched) and a college may find some students unacceptable. College preferences are *responsive* in that preferences between any two students are independent of the other students admitted by the college. Each college has a *quota*, the maximum number of students that it can admit.

Given the preferences of students and colleges, the algorithm proceeds as follows: (i) if any student s has a favorite college c amongst all acceptable colleges that have not yet filled their quota, and college c has q positions left to fill, and student s is one of the q most preferred students of college c out of all acceptable, so-far-unmatched students, then student s is matched to college c; (ii) remove any matched student, and any college whose quota is full, from the market; (iii) repeat until the condition in (i) cannot be satisfied.

The algorithm has the flavor of the deferred acceptance (DA) algorithm (also Gale and Shapley [op. cit.]), with the difference that when colleges and students are matched under DA, the matching is provisional, whereas here it is irrevocable. The authors show how this difference leads to the possibility of the algorithm terminating with unmatched students and empty college places even when the students and colleges concerned would benefit from matching. However, when this does not happen, the outcome is the same as student-proposed DA, which is known to lead to the best stable matching from the students' perspective. The theorem of the paper states that, under the circumstances just described, this is also the best stable matching from the college's perspective. As the set of stable matchings has a lattice structure with the best stable matchings for each side of the market as maximal and minimal elements, this implies a unique stable matching.

The proof in the paper uses the description of the algorithm directly. However, an alternative proof might work as follows: (i) replace each college of quota q with q new-colleges of quota 1 with the same preferences as the original college; (ii) slightly perturb students' preferences so that they have strict preference orderings over these q new-colleges, but that their preference orderings over these new-colleges with respect to new-colleges created from any other college is given by their orderings over the two original colleges; (iii) note that the problem is now symmetric: students and new-colleges are identical objects; (iv) by symmetry of the algorithm, if the best stable matching for students is selected, then the best stable matching for new-colleges is also selected; (v) note that as college preferences in the original problem are responsive, a matching is stable in this derived problem if and only if it corresponds to a stable matching in the original problem [see A. E. Roth and M. O. Sotomayor, *Two-sided matching*, Econom. Soc. Monogr., 18, Cambridge Univ. Press, Cambridge, 1990; MR1119308].

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.