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Pairwise stable matching in large economies. (English summary)
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There are two sets of types, W and M, corresponding to sets of women and men. A pair in $W \times M$ is a *couple type*. For each couple type, there is a set of available contracts. C is the set of all contracts and the *contract correspondence* $\mathcal{C}: W \times M \to 2^C$ specifies which contracts are available to each couple type. Each $w \in W$ has a strict preference relation $\succ_w$ over $M \times C$ and each $m \in M$ has a strict preference relation $\succ_m$ over $W \times C$. The full model also includes the possibility of women or men remaining unmatched, but for brevity and clarity, we omit this here. The sets W, M, and C are endowed with separable and completely metrizable topologies. Preferences are assumed acyclic and continuous, and the contract correspondence is assumed continuous with nonempty and compact values.

The distribution of types is given by v_W and v_M, which are nonzero, finite, Borel *population measures* on W and M, respectively. A matching μ is a Borel measure on $W \times M \times C$ that has v_W and v_M as the relevant marginal measures and respects that couple types only choose among contracts available to them.

Let $\mu \otimes \mu$ be the product measure of μ with itself. A matching μ is *stable* if this product measure puts zero weight on pairs of couple types and contracts that can be considered dissatisfied with μ. Such a pair could be (w, m, c) and (w', m', c') such that $(m', c'') \succ_w (m, c)$ and $(w, c'') \succ_{m'} (w', c')$ for some $c'' \in \mathcal{C}(w, m')$. Alternatively, it could be a couple type that selects an inefficient contract. That is, $(m, c'') \succ_w (m, c)$ and $(w, c'') \succ_m (w, c)$ for some $c'' \in \mathcal{C}(w, m)$.

The main theorem (Theorem 1) proves the existence of at least one stable matching. The proof considers a sequence of pairs of measures (v_W^n, v_M^n) with finite support, taking only rational values, with v_W^n converging to v_W and v_M^n converging to v_M. Existence of a stable matching μ_n for (v_W^n, v_M^n) follows from the existence of a stable matching in the corresponding finite population matching problem, which can be constructed using the assumption of finite support and rational values. It then follows from the continuity and compactness assumptions that a convergent subsequence of μ_n converges to a stable matching μ of the original problem. *Jonathan Newton*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.