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A bargaining set for roommate problems. (English summary)
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Consider a *roommate problem* in which each agent i in a finite set of agents N has a complete, transitive, strict preference ordering $\succ_i$ over N. A *matching* μ is a one-to-one function from N to N. If $\mu(i)=i$, agent i is *unmatched* at μ. Assume this is the least preferred outcome for i. Otherwise i is *matched* at μ. Players' preferences over agents extend to preferences over matchings in the natural way.

A pair of agents i,j is *mutually best* if each is the other's most preferred partner.

Matching μ is *weakly efficient* if there is no matching that every agent strictly prefers to μ.

A pair i,j is a *blocking pair* for μ if they strictly prefer one another to their partners at μ.

A blocking pair i,j is *weak* if i (respectively, j) has another blocking pair with an agent k who he strictly prefers to j (respectively, i). A matching is *weakly stable* if all blocking pairs are weak.

Theorem 1 shows that there exists a weakly stable matching. The proof is constructive, iteratively matching mutually best pairs of agents and removing them from the problem. When this is no longer possible, remaining agents are left unmatched. It is shown that the resulting matching is weakly stable.

A set of agents S can *enforce* μ' over μ if agents in S whose partners differ at the two matchings are matched within S at μ'.

An *objection* to μ is a pair (S,μ'), $S\neq\varnothing$, such that S can enforce μ' over μ and all agents in S strictly prefer μ' to μ.

A *counterobjection* to an objection (S,μ') is a pair (T,μ'') such that S and T intersect but neither contains the other, T can enforce μ'' over μ, all agents in $T\setminus S$ weakly prefer μ'' to μ, and all agents in $T\cap S$ weakly prefer μ'' to μ'.

An objection is *justified* if there does not exist any counterobjection to it.

The *bargaining set* is the set of matchings that have no justified objections.

Theorem 2 shows that the bargaining set coincides with the set of weakly stable and weakly efficient matchings. No proof is given, as the theorem straightforwardly generalizes a result of F. Klijn and J. Massó [Games Econom. Behav. **42** (2003), no. 1, 91–100; MR1968619].

Theorem 3 shows that the bargaining set is nonempty. Given Theorem 2, this reduces to showing that there exists a weakly stable matching that is also weakly efficient. If the problem includes any mutually best pairs, then matchings at which these pairs are matched must be weakly efficient. In the proof of Theorem 1, such a matching is determined and shown to be weakly stable. Therefore, by Theorem 2, we are done. Next, consider a problem that includes no mutually best pairs. If there is an odd number of agents, then any matching must have at least one agent who remains unmatched, therefore the matching at which all agents are unmatched is weakly efficient. Moreover, by Theorem 1 this matching is also weakly stable, so by Theorem 2 we are done. If there is an even number of agents, the proof is more complicated and proceeds through extensive case checking, using a decomposition result from [J. J. M. Tan, J. Algorithms **12** (1991), no. 1, 154–178; MR1088121]. *Jonathan Newton*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.