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Karpov, Alexander (RS-HSE)

A necessary and sufficient condition for uniqueness consistency in the stable marriage matching problem. (English summary)

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Consider the standard matching problem of D. Gale and L. S. Shapley [Amer. Math. Monthly **69** (1962), no. 1, 9–15; MR1531503]. There is a set of agents N comprising a finite set of men M and a finite set of women W. A matching μ is a mapping from $M \cup W$ to $M \cup W \cup \{\varnothing\}$ such that for $m \in M$, $\mu(m) \in W \cup \{\varnothing\}$; for $w \in W$, $\mu(w) \in M \cup \{\varnothing\}$; and $\mu(m) = w$ implies $\mu(w) = m$. Each man has a strict preference ordering over $W \cup \{\varnothing\}$. Each woman has a strict preference ordering over $M \cup \{\varnothing\}$. The preferences of the men and women are collected in a preference profile P.

A preference profile P satisfies *uniqueness consistency* if (i) there is a unique stable matching μ^*, and (ii) if we consider a restriction of the set of agents to $N' \subset N$ such that if $x \in N'$, then $\mu^*(x) \in N'$, together with preferences P restricted to N' to give P', then the restricted problem also has a unique stable matching.

A preference profile P satisfies the *α-condition* if there is a stable matching μ^* and (i) there exist orderings on M and F respectively such that a woman w_i in position i prefers $\mu^*(w_i)$ to any man in any position $j > i$, and (ii) there exist orderings on M and F respectively such that a man m_i in position i prefers $\mu^*(m_i)$ to any woman in any position $j > i$.

Theorem 1 states that the α-condition implies uniqueness consistency. The proof is that, in both the original problem and the restricted problems described above, the α-condition implies that the Gale-Shapley algorithm for finding a stable matching chooses the same matching under the man-proposing variant (which selects the best stable matching for men) and the woman-proposing variant (which selects the best stable matching for women). Consequently, the best stable matchings for men and women respectively are identical. That is, there exists a unique stable matching.

Theorem 2 states that if the worst outcome for any man or woman is to remain unmatched, then uniqueness consistency implies the α-condition. The proof is by induction on $|M|$ and $|W|$. Assuming that we have orderings as per the definition of the α-condition for $|M| = n_M$ and $|W| = n_W$, it is shown that these orderings can be extended for $|M| = n_M + 1$ and $|W| = n_W$. *Jonathan Newton*

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