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College admissions with entrance exams: centralized versus decentralized. (English summary)

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The paper considers a discrete college-student matching problem in which there are two colleges (indexed $1, 2$) and n students. College i can admit a quota of q_i students. It is assumed that $q_1 + q_2 \leq n$. If a student attends college i he obtains (cardinal) utility of v_i. If he does not attend college he receives utility of zero. It is assumed that every student prefers college 2 to college 1, $v_2 > v_1 > 0$. Each student has an ability (a *type*, in standard terminology), with each student's ability being drawn independently according to the same continuous distribution. Each student chooses a level of effort and suffers a disutility from doing so. This disutility of effort is lower for students with higher ability.

Two systems of matching students to colleges are considered. Firstly, a *centralized* system is considered. This is a game in which (i) the player set is the set of students; (ii) the strategy for each student is his effort choice given his ability; (iii) the payoffs are given by the utilities described above after the q_2 students with the highest effort levels are assigned to college 2 and, of the remaining students, the q_1 students with the highest effort levels are assigned to college 1. The paper finds a symmetric Bayesian Nash equilibrium (BNE) of this game. This equilibrium is the unique symmetric BNE, is in pure strategies, and is monotone in that students with higher abilities choose higher effort. The equilibrium is found using techniques, standard in auction theory, that move from continuity of the type (i.e., ability) space to continuity of the bidding (i.e., effort) function.

Secondly, a *decentralized* system is considered. This is a game in which (i) the player set is the set of students; (ii) the strategy for each student is his effort choice given his ability *and* a choice of one of the two colleges; (iii) the payoffs are given by the utilities described above when (a) of the students who chose college 2, the q_2 students with the highest effort levels are assigned to college 2 and (b) of the students who chose college 1, the q_1 students with the highest effort levels are assigned to college 1. The paper finds a symmetric and monotone BNE of this game. It is the unique symmetric and monotone BNE. This equilibrium has students with ability above some cutoff value choosing college 2 for sure, and the students with ability below this cutoff value randomizing between college 1 and college 2. That is, in the equilibrium considered, the students below the cutoff value play mixed strategies.

The reason (page 926, line 4, some argument omitted) that there is no symmetric, monotone equilibrium in pure strategies under the decentralized system is that a player at the cutoff value would have to be indifferent between choosing the two colleges (plus whatever effort levels are optimal for each college). If he were to choose college 2, his optimal effort must be zero in a monotone equilibrium as all players with higher abilities than his own will choose a higher effort by construction, so he will only get into college 2 when there are sufficiently few of these players. If he were to choose college 1, then his optimal effort must be strictly positive, as if it were zero, then he would have some incentive to distinguish himself from other players applying to college 1 by some infinitesimal increase in effort, just in case more than q_1 players choose college 1 (note that this argument relies upon $q_1 < n$). This effort discontinuity then (page 926, line 5 onward) creates incentives for deviation and violates equilibrium conditions.

The paper lacks a good justification for allowing mixed strategies. Mixed strategies can be considered as an abstraction from asymmetric equilibria in pure strategies, but in this case it would be nice to see some stability analysis, such as testing for evolutionary stability or robustness to random shocks [J. Newton and R. Sawa, J. Econom. Theory **157** (2015), 1–27; MR3335933; B. Klaus and J. Newton, J. Math. Econom. **62** (2016), 62–74; MR3435745; H. H. Nax and B. S. R. Pradelski, Internat. J. Game Theory **44** (2015), no. 4, 903–932; MR3422869]. The paper leaves open the question of what asymmetric equilibria exist. *Jonathan Newton*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.