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Game theoretical modelling of a dynamically evolving network II: Target sequences of score 1. (English summary)

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This paper considers simple (undirected, unweighted, no multiple edges) graphs with vertex set V. The vertex set V represents a set of *players*. Each player has a *target* number of other players with whom he would like to share an edge. The target of player i is denoted t_i.

For a given graph, let e_i be the degree of player/vertex i. The *deviation* of player i is then given by $|t_i - e_i|$. The deviation of the graph is the sum of the deviations of all players. The minimum deviation over all possible graphs is the *score*. $K(\min)$ denotes the set of graphs that attain this minimum deviation. Note that the lowest possible score of zero implies that every player attains his target degree.

For any graph, a vertex can be one of three types. A *joiner* has $e_i < t_i$ and therefore wishes to increase his degree. A *breaker* has $e_i > t_i$ and therefore wishes to decrease his degree. A *neutral* player has $e_i = t_i$ and therefore wishes to keep his degree the same.

Theorem 3.1 shows that the score equals one if and only if there exists a sequence $(g_i)_{i\in V}$ such that $|t_i - g_i| = 1$ for some i and $t_j = g_j$ for all $j \neq i$. In words, every set of targets that gives a score of one is close (in an obvious sense) to a set of targets that gives a score of zero.

Given the players' targets, vertices can be categorized into four types: (A) vertices which are joiners for some graphs in $K(\min)$ and breakers for some graphs in $K(\min)$; (J) vertices which are never breakers in $K(\min)$; (B) vertices which are never joiners in $K(\min)$; (N) vertices which are always neutral in $K(\min)$.

The main theorem of the paper (Theorem 3.3) considers initial targets with score one and nearby (in the sense of Theorem 3.1) targets with score zero. The types of changes (for example, adding or subtracting one from the targets of vertices with high or low targets) required to obtain t_i (the score one targets) from g_i (the score zero targets) are then shown to characterize the four classes of vertices defined above.

{For Part I see MR3716169.} *Jonathan Newton*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.