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Sequential versus simultaneous assignment systems and two applications. (English summary)
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Consider a many-to-one matching problem consisting of a finite set of *students* and a finite set of *colleges*, each of which has the *capacity* to admit up to a given number of students. Each student has a preference ordering over the colleges and over the possibility of being *unassigned* (i.e., having no college). Each college has a preference ordering over students. The preference of a college between any two students does not depend on the other students that attend the college. A college may prefer to leave some places unassigned rather than admit certain students.

A *matching* is a mapping from students to colleges such that no college is assigned more students than its capacity. A matching is *non-wasteful* if there does not exist a college with an unassigned place such that (i) a student would prefer this college to the college (or no college) that he or she is assigned under the matching, and (ii) the college would prefer to admit this student rather than leave the place unassigned. A matching is *individually rational* if (i) every student prefers his or her assigned college to remaining unassigned, and (ii) every college prefers each of its assigned students to an unassigned place.

Given a many-to-one matching problem, a *mechanism* maps *stated preference* orderings submitted by the students to matchings in the respective problem. Stated preferences may differ from the true preferences of the students. The paper considers *sequential* mechanisms: first, a subset of colleges is considered and students are assigned to these colleges according to a sub-mechanism; second, the remaining colleges are considered and students are assigned to these colleges according to another sub-mechanism. Stated preferences may differ for each sub-mechanism.

The main theorems of the paper consider the case in which each sub-mechanism is similar to a known non-sequential mechanism and give results concerning Nash equilibrium (NE) of the game in which the strategy of each student comprises his or her stated preference orderings. Here, we restrict discussion to the case in which each sub-mechanism is the deferred acceptance (DA) mechanism.

Theorem 1 shows that the NE of the game then leads to non-wasteful and individually rational matchings. Individual rationality follows trivially from the definition of NE. To see non-wastefulness, conjecture that college X has an unassigned place at some matching arising from NE, student Y prefers college X to his assignment at this matching and college X prefers student Y to leaving the place unassigned. We show that this cannot be an NE. If college X is considered by the first sub-mechanism, then student Y could guarantee a place in college X by ranking college X as his or her first preference. If college X is considered by the second sub-mechanism, then student Y could guarantee a place in college X by ranking remaining unassigned as his or her first preference for the first sub-mechanism and ranking college X as his or her first preference for the second sub-mechanism. The reason this would work is that, under DA, effectively opting out of assignment at the first stage does not cause any other students who were previously assigned at the first stage to become unassigned at the first stage. That is, opting out of the first stage does not increase the competition for a

place at college X in the second stage. The argument is complete.

More general results and converse results are also given. *Jonathan Newton*