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Speed limits. (English summary)
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There are two players, an agent and a regulator. The agent chooses an activity level $x \in [0, \overline{x}]$. Prior to the agent choosing an activity level, the regulator chooses, for each activity level x that the agent could choose, a probability that an agent choosing that activity level will be subject to a fine. The fine is a fixed amount. The probability that an agent pays a fine is assumed weakly increasing in x. The agent has a continuous increasing utility function $u(\cdot)$. If he chooses x and pays no fine, his utility is $\tilde{\alpha}u(x)$. If he pays a fine, his utility is $\tilde{\alpha}u(x) - F$.

The agent's parameter $\tilde{\alpha}$ can take one of two values, α_L (low type) or α_H (high type). Intuitively, the high type agent gains more from undertaking more of the activity x. The regulator does not know whether the agent is low or high type, so cannot condition his strategy on the agent's type.

The regulator prefers lower amounts of x and wishes to minimize (in expectation) the strictly increasing and continuous loss function $l(x)$.

It is assumed that $F < \alpha_L\,(u(\overline{x}) - u(0))$, so that if the agent is faced with the prospect of paying the fine whenever he undertakes any strictly positive level of activity, then he will choose the maximum level of activity $x = \overline{x}$.

Assume that the agent optimizes given the regulator's choice. The main theorem (Theorem 1) states that if $l(\cdot)$ is convex, the marginal utility of activity $\frac{\partial u(x)}{\partial x} \to 0$ as $x \to \infty$, the probability that the agent is a low type is large enough, and $\overline{x}$ is large enough, then it is optimal for the regulator to choose two thresholds y' and y'', such that for $x \leq y'$, the agent pays no fine; for $y' < x \leq y''$, the agent pays the fine with some probability q, $0 < q < 1$; and for $x > y''$, the agent pays the fine with certainty. Under this optimal policy, the low type agent chooses $x = y'$ and the high type agent chooses $x = y''$.

The proof involves two propositions. Proposition 1 shows that for any non-deterministic strategy that the regulator might choose, there is a two-threshold strategy that does just as well in terms of minimizing activity. This construction involves, for a given strategy of the regulator, setting y' equal to the smallest value of x that is a best response for a low type (the reason the smallest value is specified is that there may be multiple best responses), and setting y'' equal to the smallest value of x that is a best response for a high type.

It is clear how the convexity and marginal utility assumptions of the theorem statement relate to interior solutions. Proposition 2 uses these assumptions to show that two-threshold strategies improve upon deterministic one-threshold strategies in which a fine is paid (respectively, not paid) with certainty whenever the threshold is exceeded (respectively, not exceeded).

The proof of Theorem 1 is then completed by showing that an optimal two-threshold strategy exists. *Jonathan Newton*