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Stable matching in large economies. (English summary)

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This paper considers a firm-worker matching problem. There are a finite number of firms and a continuum of workers. Each worker can be matched to at most one firm. Each worker has a type. Each worker has a strict preference ranking over firms. All workers of a given type have the same preference ranking. Each firm has a choice correspondence which gives the measure of workers of each type that it would accept from any set of available workers.

Consider a matching such that each firm f is matched to workers M_f. The matching is *stable* if it is individually rational and there does not exist a *blocking coalition*: a firm f and a set of workers M'_f such that (i) every worker in M'_f is either already matched to f or prefers f to their current firm, and (ii) if the set of available players is given by the combination of M_f and M'_f, then the choice correspondence of f includes M'_f but does not include M_f.

The main result of the paper is to give conditions under which a stable matching exists. Firstly, note that a stable matching can, by definition, be considered as a fixed point of a correspondence (in fact there are several such correspondences that can work). The paper under consideration chooses a correspondence that is similar to the deferred acceptance (DA) algorithm of D. Gale and L. S. Shapley [Amer. Math. Monthly **69** (1962), no. 1, 9–15; MR1531503]. However, the possibility of complementarities in a firm's choice correspondences (e.g., a firm wishes to recruit type A workers, but only at a one-to-one ratio with type B workers) means that the algorithm may not converge.

Conditions on the correspondence that guarantee a fixed point are similar to those used in most equilibrium existence theorems in game theory. Specifically, upper hemi-continuity and convex-valuedness of the firms' choice correspondences are assumed and the DA-style correspondence inherits these properties. The Kakutani-Glicksberg-Fan fixed point theorem then gives the existence of a stable matching. *Jonathan Newton*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.