MR3758829 91A43 05C57 91A22
Duffy, Christopher (3-SK-MS);
Janssen, Jeannette [**Janssen, Jeannette C. M.**] (3-DLHS-MS)

The spread of cooperative strategies on grids with random asynchronous updating. (English summary)

Internet Math. **2017,** 21 *pp.*

In the paper under review, Duffy and Janssen consider a game played by players represented by vertices on a specific graph: the two-dimensional square lattice with von Neumann neighborhoods wrapped on a torus. Each player has two strategies, *cooperate* and *defect*. A player obtains payoffs from playing his current strategy against each of his neighbors on the graph. His total payoff is then the sum of his payoffs from playing against each of his neighbors. Either strategy played against defect gives a payoff of zero. Cooperate played against cooperate gives a payoff of 1. Defect played against cooperate gives a payoff of $T > 1$. These payoffs differ from the classic prisoners' dilemma only in that there is no strict incentive to defect against a defecting opponent.

The paper considers a strategy updating process whereby players may, one at a time, change their strategy to imitate the player in their neighborhood who is currently obtaining the highest total payoff. The process is first studied for initial strategy profiles in which a small cluster of players cooperates or defects and every other player plays the alternative strategy. These initial results are then used to prove results for the spread of cooperation on large graphs (of size n) when the process is initialized with each player independently starting as a cooperator with probability p and as a defector with probability $1-p$. Consideration is given to both very small values of p, so that attention can be restricted to isolated clusters of a few cooperators, and very small values of $1-p$, so that attention can be restricted to isolated defectors.

The main theorem (Theorem 19) gives the amount of cooperation that can be expected asymptotically almost surely (as $n \to \infty$) for $p = f(n)$ such that asymptotically almost surely the process at initialization (1) has no clusters of three or more cooperators; or (2) has clusters of three cooperators but no clusters of four or more cooperators; or (3) has clusters of three and four cooperators but no clusters of five or more cooperators; or (4) has isolated defectors, but no clusters of two or more defectors; or (5) has no defectors.

On page 12, the notation $\ll$ is defined as $f \ll g$ if $\frac{f(n)}{g(n)} \to 0$ as $n \to \infty$. The conditions on p in the statement of (4) and (5) in the main theorem appear to be inconsistent with this definition. The reader may wish to interpret these conditions as I did in terms of $1-p$, specifically the condition for (4) as $n^{-2} \ll 1-p \ll n^{-1}$ and the condition for (5) as $1-p \ll n^{-2}$. The conditions given in the statement of the theorem appear to have been obtained as rearrangements of these, which is not permissible under the given definition of $\ll$.

A final point to note is that the authors sometimes write 'collaborators' instead of 'cooperators'. It is standard terminology to use 'cooperator' to mean a player choosing the strategy cooperate in a prisoner's dilemma. 'Collaborator', on the other hand, has been used to describe collaborative strategic choice, as distinct from cooperation. See, for example, [J. Newton, Games Econom. Behav. **104** (2017), 517–534; MR3681061], particularly Section 2.6, in which players collaborate to cooperate in a prisoners' dilemma. Also relevant to the current paper is the collaboration on square lattices with von Neumann neighborhoods in coordination games (not prisoners' dilemmas) found in [J. Newton and S. D. Angus, J. Econom. Theory **157** (2015), 172–187; MR3335940].

Jonathan Newton